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Hale and Lin have studied extensively the problem of transverse homoclinic orbits of periodic orbits of functional differntial equations (FDE's). They have shown that the classical symbolic dynamics for such problems in finite dimension also holds for FDE's (Hale and Lin [1]). These results were applied to two examples that had previously been cindifered by Walther and an der Heiden (Hale and Lin [2]). For these examples, it was a transverse homoclinic orbit to a periodic orbit and, thus "chaos" occurs and is persistent under perturbations of thr vector field. The latter important property could be obtained by the methods of Walther and an der Heiden.

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# AFOSR FINAL SCIENTIFIC PROGRESS REPORT FOR YEAR 1984-87

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Jack K. Hale

The thrust of the research fell into three categories.

1. **Chaos in delay equations.**

Hale and Lin have studied extensively the problem of transverse homoclinic orbits of periodic orbits of functional differential equations (FDE's). They have shown that the classical symbolic dynamics for such problems in finite dimension also holds for FDE's (Hale and Lin [1]). These results were applied to two examples that had previously been considered by Walther and an der Heiden (Hale and Lin [2]). For these examples, it was shown that there was a transverse homoclinic orbit to a periodic orbit and, thus, "chaos" occurs and is persistent under perturbations of the vector field. The latter important property could be obtained by the methods of Walther and an der Heiden.

If there is a homoclinic orbit to a periodic orbit for an FDE, it is important to know if there is a small perturbation of the vector field which will lead to a transverse homoclinic orbit. This was shown to be true by Hale and Lin [3]. The methods are new even for the finite dimensional case.

Hale and Sternberg [11] haave devised a new numerical scheme for determining if chaos is the result of the creation of a transverse homoclinic orbit for an unstable periodic orbit. This was applied to a specific type of equation for which chaos had been previously observed.

2. **Varying diffusivity and boundary conditions in reaction-diffusion equations.**

Hale and Rocha have been studying extensively systems of reaction diffusion equations and attempting to understand the effects of boundary conditions on the

flow when the diffusion coefficients and the boundary conditions are varied. Hale [4] has extended the results of Conway, Hoff and Smoller on large diffusivity and Neumann conditions to allow situations where invariant regions do not exists. For large diffusivity, Hale and Rocha [5], [6] have studied the effects of variations in the boundary conditions on the flow on the attractor. These results clarify the concept of well-mixed when the boundary conditions are not of Neumann type. These results also show how the flow changes as one goes from Neumann to Dirichlet conditions.

Hale and Sakamoto [12] have studied the effects of boundary conditions and diffusion coefficients on the flow when only some of the diffusion coefficients are larger.

3. **Singularly perturbed problems.** Hale and Raugel [9], [14] have studied extensively the relationships between the attractor for a singularly perturbed hyperbolic equation and the corresponding one for the limiting parabolic one. These results give a method for the comparison of flows for hyperbolic and parabolic equations on the relevant part of the flow; namely, the flow on the attractor.

Hale and Sakamoto [13] have begun the study of the existence and stability of transition layers in singularly perturbed parabolic equations uses the methods of dynamical systems. This method extends, unifies, clarifies and simplifies previous results.

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[3] J. K. Hale and X.-B. Lin, "Heteroclinic orbits for retarded functional differential equations," J. Differential Equations, 65(1986), 175-202.

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[7] J. K. Hale, Some examples of infinite dimensional systems. Contemporary Math. 58(1987), Part III, 173-182.

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Jack K. Hale has completed two manuscripts:

[9] (with G. Raugel) Upper semicontinuity of the attractor for a singularly perturbed hyperbolic equation. J. Differential Equations 73 (1988), 197-214.

[10] (with N. Stavrakakis) Compact attractors for weak dynamical systems. Applicable Analysis 26 (1988), 271-287.

[11] Onset of chaos in differential delay equations (with N. Sternberg) Computational Physics, July, 1988.

[12] Shadow systems and attractors in reaction-diffusion equations (with K. Sakamoto) *LCDS/CCS* #87-28

[13] Existence and stability of transition layers (with K. Sakamoto) LCDS/CCS #87-27. To appear in Japan J. Appl. Math.

[14] Lower semicontinuity of the attractor for a singularly perturbed hyperbolic equation. LCDS/CCS #88- .